Section 7: #1–33 EOO & 34–40 all

(1) $\frac{1}{1} + \frac{1}{16} + \frac{1}{81} + \frac{1}{256} \ldots\ldots$

$1^4 \quad 2^4 \quad 3^4 \quad 4^4$

$\sum_{n=1}^{\infty} \frac{1}{n^4} \Rightarrow$ Converges by P-Series Test $4>1$

(5) $\sum_{n=1}^{\infty} \frac{3}{n^{1/5}} \Rightarrow$ Diverges by P-Series Test $1/5 < 1$

(9) $\sum_{n=1}^{\infty} \frac{3n^{1/3}}{2n^{2/5}} = \sum_{n=1}^{\infty} \frac{3}{2}\left(\frac{n^{5/15}}{n^{6/15}}\right) =$

$\frac{3}{2}\sum_{n=1}^{\infty} \frac{1}{n^{1/15}} \Rightarrow$ Diverges by P-Series Test $1/15 < 1$

(13) $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$ Since $\frac{1}{n} < \frac{1}{\ln(\ln n)}$

and $\sum_{n=3}^{\infty} \frac{1}{n}$ Diverges

then $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)} \Rightarrow$ Diverges by Direct Comparison Test

(17) $\sum_{n=2}^{\infty} \frac{n^3-1}{n^4+1}$ VS. $\sum_{n=2}^{\infty} \frac{n^3}{n^4}$

Since $\frac{n^3-1}{n^4+1} < \frac{n^3}{n^4}$ and $\sum_{n=2}^{\infty} \frac{1}{n}$ Diverges

Direct Comparison Test won't work!

Do Limit Comparison!

$\lim_{n\to\infty} \frac{n^3-1}{n^4+1} \cdot \frac{n^4}{n^3} = 1$

So, $\sum_{n=2}^{\infty} \frac{n^3-1}{n^4+4}$ also Diverges

(21) $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\ldots$

$$\sum_{n=1}^{\infty}\frac{1}{2n} \quad \text{vs.} \quad \sum_{n=1}^{\infty}\frac{1}{n}$$

$$\lim_{n\to\infty}\frac{1}{2n}\cdot\frac{n}{1}=\frac{1}{2} \Rightarrow \text{positive \& finite}$$

Since $\sum_{n=1}^{\infty}\frac{1}{n}$ Diverges, $\sum_{n=1}^{\infty}\frac{1}{2n}$ DIVERGES by the Limit Comparison Test

(25) $\sum_{n=1}^{\infty}\sin(1/n)$

*Use limit comparison test!

$$a_n=\sin(1/n) \quad \text{vs.} \quad b_n=1/n$$

$$\lim_{n\to\infty}\frac{\sin(1/n)}{1/n}\overset{H}{=}\lim_{n\to\infty}\frac{-\frac{1}{n^2}\cos(1/n)}{-1/n^2}=$$

$$\lim_{n\to\infty}\cos(1/n)=1$$ *this is positive & finite

So since $\sum_{n=1}^{\infty}\frac{1}{n}$ Diverges,

$\sum_{n=1}^{\infty}\sin(1/n)$ also DIVERGES

(29) $\sum_{n=1}^{\infty}\frac{1}{n}\sin(1/n)$ *Use limit comparison test

$$\begin{cases} a_n=\frac{1}{n}\sin\left(\frac{1}{n}\right) \\ b_n=\frac{1}{n^2} \end{cases}$$

$$\lim_{n\to\infty}\frac{\frac{1}{n}\sin\left(\frac{1}{n}\right)}{\frac{1}{n^2}}=\lim_{n\to\infty}\frac{\sin\left(\frac{1}{n}\right)}{1/n}=1 \text{ (see \#25)}$$

*Since $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges, $\sum\frac{1}{n}\sin(1/n)$ also CONVERGES

(33) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

* Limit comparison test

$a_n = \frac{e^{1/n}}{n^2}$ $b_n = \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{e^{1/n}}{\cancel{n^2}} \cdot \frac{\cancel{n^2}}{1} =$

$\lim_{n\to\infty} e^{1/n} = e^0 = 1$

* Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ CONVERGES by P-Series Test.

$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ converges too by the Limit comparison Test.

(34) Integral Test!

$\sum_{n=0}^{\infty} \frac{1}{2n+1}$

$\lim_{b\to\infty} \int_0^{\infty} \frac{1}{2x+1}\,dx = \lim_{b\to\infty} \left[\frac{1}{2}\ln|2x+1|\right]_0^b =$

$\lim_{b\to\infty} \frac{1}{2}\ln|2b+1| - \cancel{\frac{1}{2}\ln 1} = \boxed{\infty}$

So series DIVERGES

(35) Nth term test!

$\sum_{n=1}^{\infty} n$ $\lim_{n\to\infty} n \neq 0$, so series DIVERGES

(36) P-series test!

$\sum_{n=1}^{\infty} \frac{1}{n^{1/2}} \Rightarrow$ DIVERGES since $1/2 < 1$

(37) Geometric Series Test!

$$\sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n \Rightarrow \text{Converges since } 3/4 < 1$$

(38) Ratio Test!

$$\sum_{n=0}^{\infty} \frac{n!}{3^n} \qquad \lim_{n\to\infty} \frac{(n+1)!}{\cancel{3}^{n+1}} \cdot \frac{\cancel{3}^n}{n!} =$$

$$\lim_{n\to\infty} \frac{n+1}{3} = \infty > 1$$

So series DIVERGES

(39) Limit Comparison Test!

$$a_n = \frac{3}{n^2-1} \qquad b_2 = \frac{3}{n^2}$$

$$\lim_{n\to\infty} \frac{\cancel{3}}{n^2-1} \cdot \frac{n^2}{\cancel{3}} = 1$$

Since $\sum_{n=2}^{\infty} \frac{3}{n^2}$ Converges by p-series test

$\sum_{n=2}^{\infty} \frac{3}{n^2-1}$ CONVERGES too!

(40)

Direct Comparison Test!

$$\sum_{n=1}^{\infty} \frac{e^n}{n} \qquad \frac{e^n}{n} > \frac{1}{n}$$

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ Diverges by p-series test,

then $\sum_{n=1}^{\infty} \frac{e^n}{n}$ DIVERGES too!